

# Coherent radiation of atoms and a channeling particle

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## Abstract

New mechanism of radiation emitted at channeling of a relativistic particle in a crystal is studied. Superposition of coherent radiation of the atoms in a crystal lattice which are excited by a channeling particle and radiation of the channeling particle itself is considered. It is shown that coherent radiation of the chain of oscillating atoms forms a resonance peak on the background of radiation of the channeling particle.

*Keywords:* radiation, channeling, crystal lattice, atoms vibration, coherence

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<sup>1</sup> **1. Introduction**

<sup>2</sup> Channeling of the charged particles was predicted by American scientists M.T. Robinson and O.S. Oen in 1961 [1] and was discovered by several groups of scientists in 1963 –  
<sup>3</sup> 1965. Channeling of the accelerated particles was investigated in detail both theoretically  
<sup>4</sup> and experimentally. The effect of channeling has served as the base for development of  
<sup>5</sup> new experimental methods of research on the crystal structure. It makes possible to  
<sup>6</sup> measure distribution of electronic density in inter-atomic space of crystals and exact ori-  
<sup>7</sup> entation of the crystal planes. Channeling of ions is used recently for direct detections  
<sup>8</sup> of WIMP (Weakly Interacting Massive Particle) – the Dark Matter candidates [2, 3].

<sup>10</sup> At interaction between the channeling particle and a crystal the electromagnetic  
<sup>11</sup> radiation is generated. In the framework of classical electrodynamics one can consider  
<sup>12</sup> this radiation as radiation from different sources, such as radiation of the channeling  
<sup>13</sup> particle, radiation of the electronic gas (wake trace) and radiation of the atoms excited  
<sup>14</sup> by a channeling particle. Radiation of a channeling particle is investigated in details  
<sup>15</sup> in many papers and books, see for example [4, 5]. Experimental study of radiation  
<sup>16</sup> of wake fields can be found in [6]. Less attention is paid to radiation of the atoms  
<sup>17</sup> excited by a channeling particle. The last effect consists of two different parts – radiation  
<sup>18</sup> emitted at decay of exited state of atom, and radiation of vibrating atomic nucleus.  
<sup>19</sup> When channeling, the charged particle transmits part of its transverse momentum to  
<sup>20</sup> the surrounding atoms. This excites vibration of atomic nucleons together with the  
<sup>21</sup> inner electrons. This phenomenon is similar to the vibrational excitation of molecules  
<sup>22</sup> and causes respective radiation. But vibrations of atoms in the channel are coherent,

23 because they are excited by the same channeling particle. This can lead to sufficient  
 24 amplification of radiation intensity .

25 Assumption on radiation of this kind was expressed recently in [7]. It was shown that  
 26 radiation of an atomic chain is collimated in direction of the channeling particle velocity.  
 27 But proper radiation of the channeling particle was not taken into account. Actually,  
 28 the field of radiation should be calculated as a superposition of fields generated both by  
 29 channeling particle and excited atoms.

30 In this paper we study the field of vibrating atoms in some more details and calcu-  
 31 late angular distribution of radiation resulting from superposition of fields produced by  
 32 channeling particle and by atoms. In section 2 we discuss some features of radiation  
 33 field of atomic chain which were not studied in [7]. Next section presents calculation of  
 34 the net field produced by channeling particle and excited atoms. We discuss the ratio  
 35 of radiation intensity caused by vibrating atoms and by channeling particle. Angular  
 36 distribution of radiation is plotted.

## 37 2. Field of the atomic chains exited by a channeling particle

38 Electric field  $E_a(t)$  of vibrating atoms in far zone is given by equation (30) in [7]. We  
 39 rewrite this equation in a complex form

$$E_a(t) = \frac{A_0 \nu^2}{2k} \left\{ \frac{e^{i\nu t} - e^{-i\Omega' t}}{\nu + \Omega'} + \frac{e^{i\nu t} - e^{i\Omega' t}}{\nu - \Omega'} \right\}, \quad (1)$$

40 for  $t \leq Lk$  and

$$E_a(t) = \frac{A_0 \nu^2 e^{i\nu t}}{2k} \left\{ \frac{1 - e^{-ikL(\nu + \Omega')}}{\nu + \Omega'} + \frac{1 - e^{-ikL(\nu - \Omega')}}{\nu - \Omega'} \right\}, \quad (2)$$

41 for  $t > Lk$ . Here

$$A_0 = \frac{2eq^2 K \sin \theta}{mVD^2 \omega b R c^2}, \quad k = \frac{1}{V}(1 - \beta \sin \theta \sin \varphi), \quad \nu = \omega + i\alpha, \quad (3)$$

42  $V$  is the mean velocity of the channeling particle,  $\beta = V/c$ ,  $c$  is the speed of light,  $e$   
 43 is the charge of the channeling particle,  $m$  is the mass of atom,  $2D$  is the width of the  
 44 channel,  $b$  is the spacing between atoms in an atomic chain and  $q$  is an effective charge of  
 45 a nucleus of atom which is screened by the interior electrons. Angles  $\theta$  and  $\varphi$  of spherical  
 46 coordinate system are depicted in Fig. 1  $K$  and  $\Omega$  are the amplitude and frequency of  
 47 channeling particle oscillations which are bound to  $XOY$  plane. The mean velocity of the  
 48 particle is directed along the  $X$ -axis, the particle enters the crystal at  $x = 0$  and leaves  
 49 it at  $x = L$ . An excited atom of the crystal lattice is oscillating along the  $Y$ -axis with  
 50 frequency  $\omega$  and amplitude decreasing in time as  $\exp(-\alpha t)$ , where  $\alpha$  is the attenuation  
 51 factor.

52 Further we consider only this part of radiation. Taking the real part of equation (1)  
 53 we obtain

$$E_a(t) = -\frac{A_0 \omega}{k [(\alpha^2 + \omega^2 - \Omega'^2)^2 + 4\alpha^2 \Omega'^2]} \left\{ 2\alpha \Omega'^3 \sin \Omega' t + \cos \Omega' t \left[ (\alpha^2 + \omega^2)^2 + \Omega'^2 (3\alpha^2 - \omega^2) \right] \right\}. \quad (4)$$

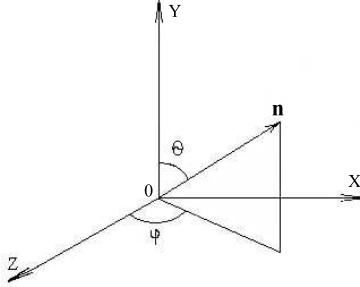


Figure 1: The coordinate system.

54 This field has some features typical for the field of relativistic charged particle though the  
 55 radiation is caused by non-relativistic vibrating atoms. First, the multiple  $1 - \beta \sin \theta \sin \varphi$   
 56 in denominator of  $A_0$  shows that main part of radiation is concentrated around the  
 57 direction of the particle velocity  $\beta$ . Second, the frequency of radiation  $\Omega'$  is just the  
 58 same as the frequency of the relativistic channeling particle. Coherence of radiation  
 59 is expressed by factor  $b$  in denominator of  $A_0$ . It shows that intensity of radiation is  
 60 proportional to the square of linear charge density  $q/b$ . Denominator of this expression  
 61 is of typical resonant character. Indeed, if attenuation of atomic vibrations vanishes  
 62 ( $\alpha \rightarrow 0$ ) and  $\Omega' = \omega$ , the electric field defined by the above equation tends to infinity.

### 63 3. Superposition with the field of channeling particle

64 Radiation of the vibrating atoms is emitted at the same frequency  $\Omega'$  as the radiation  
 65 of the channeling particle. And in case of relativistic particle it is concentrated in the  
 66 same narrow cone around the direction of the particle velocity. Hence, the fields of both  
 67 sources are correlated and the resulting field must be calculated as superposition of the  
 68 fields.

69 Let us calculate the radiation field of the channeling particle. Electric field  $\mathbf{E}_c$  of a  
 70 moving charge  $e$  is given by equation [8]:

$$\mathbf{E}_c = \frac{e}{c^2 R (1 - \beta n)^3} [\mathbf{n} [(\mathbf{n} - \beta) \dot{\mathbf{v}}]], \quad (5)$$

71 where  $\mathbf{v}$  is the velocity of charged particle and  $\beta = \mathbf{v}/c$ . Next, we assume that the  
 72 transversal component  $v_\perp$  of the particle velocity is small enough to satisfy inequality  
 73  $v_\perp \ll c\sqrt{1 - \beta^2}$ . It is shown in [8] that in this case we can consider  $\mathbf{v}$  as being constant,  
 74 keeping as time dependent only  $\dot{\mathbf{v}}$ . Hence, we put  $\mathbf{v} = (V, 0, 0)$ .

75 Substituting the law of motion  $x(t) = Vt$ ,  $y(t) = K \cos \Omega t$ ,  $z(t) = 0$  into (5) we get  
 76 the electric field components in spherical coordinates defined by Fig. 1

$$E_{c\theta} = \frac{e K \Omega^2 (\beta \sin \varphi - \sin \theta) \cos \Omega' t}{c^2 R (1 - \beta \sin \theta \sin \varphi)^3}, \quad E_{c\varphi} = \frac{e K \Omega^2 \beta \cos \theta \cos \varphi \cos \Omega' t}{c^2 R (1 - \beta \sin \theta \sin \varphi)^3}. \quad (6)$$

77 The net radiation field  $\mathbf{E}$  is a vector sum of the fields of atomic chains (4) and of the  
 78 channeling particle (6). The electric field  $\mathbf{E}_a$  is polarized in a plane which includes axis

<sup>79</sup>  $OY$ . In order to compute the intensity of radiation we average over time the square of  
<sup>80</sup> the resulting field. This yields

$$\begin{aligned}\overline{E_\theta^2} &= E_0^2 \left\{ \frac{(\beta \sin \varphi - \sin \theta)^2}{(1 - \beta \sin \theta \sin \varphi)^4} \right. \\ &\quad - 2\eta \gamma^2 \frac{\sin \theta (\beta \sin \varphi - \sin \theta) \left[ (\alpha^2 + \omega^2)^2 + \Omega'^2 (3\alpha^2 - \omega^2) \right]}{(1 - \beta \sin \theta \sin \varphi)^2 [(\alpha^2 + \omega^2 - \Omega'^2)^2 + 4\alpha^2 \Omega'^2]} \\ &\quad \left. + \eta^2 \gamma^4 \frac{\sin^2 \theta \left[ (\alpha^2 + \omega^2)^2 + 4\alpha^2 \Omega'^2 \right]}{[(\alpha^2 + \omega^2 - \Omega'^2)^2 + 4\alpha^2 \Omega'^2]} \right\},\end{aligned}\quad (7)$$

$$\overline{E_\varphi^2} = E_0^2 \frac{\beta^2 \cos^2 \theta \cos^2 \varphi}{(1 - \beta \sin \theta \sin \varphi)^4}, \quad (8)$$

<sup>81</sup> where

$$E_0 = \frac{e\Omega^2 K}{\sqrt{2} c^2 R (1 - \beta n_x)},$$

<sup>82</sup>

$$\eta = \frac{2q^2}{m\gamma^2 \Omega^2 b D^2}, \quad \gamma = \frac{1}{\sqrt{1 - \beta^2}}.$$

<sup>83</sup> Factor  $\eta$  shows the relation between the intensity of radiation generated by atoms and  
<sup>84</sup> radiation emitted by the channelling particle.

<sup>85</sup> For an ultra-relativistic particle we have only to consider small angles around the  
<sup>86</sup> velocity direction and can make the usual approximation, leading to reduced angles

$$\psi_y = \gamma \left( \frac{\pi}{2} - \theta \right), \quad \psi_z = \gamma \left( \frac{\pi}{2} - \varphi \right), \quad \psi^2 = \psi_y^2 + \psi_z^2,$$

<sup>87</sup> In this approximation (7) and (8) take the form:

$$\begin{aligned}\overline{E_\theta^2} &= E_0^2 \gamma^4 \left\{ 4 \frac{(1 - \psi_y^2 + \psi_z^2)^2}{(1 + \psi^2)^4} \right. \\ &\quad + 4\eta \frac{(1 - \psi_y^2 + \psi_z^2)^2 \left[ (\tilde{\alpha}^2 + 1)^2 + \tilde{\Omega}'^2 (3\tilde{\alpha}^2 - 1) \right]}{(1 + \psi^2)^2 [(\tilde{\alpha}^2 + \omega^2 - \tilde{\Omega}'^2)^2 + 4\tilde{\alpha}^2 \tilde{\Omega}'^2]} \\ &\quad \left. + \eta^2 \frac{(\tilde{\alpha}^2 + 1)^2 + 4\tilde{\alpha}^2 \tilde{\Omega}'^2}{[(\tilde{\alpha}^2 + \omega^2 - \tilde{\Omega}'^2)^2 + 4\tilde{\alpha}^2 \tilde{\Omega}'^2]} \right\},\end{aligned}\quad (9)$$

$$\overline{E_\varphi^2} = 32E_0^2 \frac{\gamma^4 \psi_y^2 \psi_z^2}{(1 + \psi^2)^4}, \quad (10)$$

<sup>88</sup> where

$$\tilde{\alpha} = \alpha/\omega, \quad \tilde{\Omega}' = \Omega'/\omega. \quad (11)$$

89 Let us estimate the value of  $\eta$ . The frequency  $\Omega$  depends on the continuous interaction  
90 potential of atomic strings with the channeled particle and is of order  $\Omega \approx c\psi/D$ , where  
91  $\psi$  is the Lindhard angle

$$\psi = \sqrt{\frac{eq}{\mathcal{E}b}}, \quad (12)$$

92  $\mathcal{E} = m_e c^2 \gamma$  is the energy of the channeling particle,  $m_e$  is its mass. Substituting all this  
93 into expression for  $\eta$  we get

$$\eta \approx \frac{qm_e}{em\gamma}.$$

94 If the channeling particle is a proton, the charge of nucleus is  $q = Ze$  and mass of nucleus  
95 is  $m = 2Ze$ , then  $\eta \approx \gamma^{-1}$ . But we have to take into account the resonant character of  
96 the last term in equation (9). Substituting  $\tilde{\Omega}' = 1$  into (9) we see that the ratio of last  
97 term to the first one is of order  $\eta^2 \alpha^{-2} \sim [m_e/(m\gamma\alpha)]^2$ .

98 Hence, in case of heavy channeling particle and low attenuation the radiation intensity  
99 of the atomic string can be comparable with radiation intensity of the channeling particle.  
100 The resonance condition

$$\tilde{\Omega}' = 2\gamma^2 \frac{\Omega}{\omega(1 + \psi^2)} = 1 \quad (13)$$

101 can be fulfilled only if  $2\gamma^2 \Omega / \omega > 1$ .

102 As an example we have plotted angular distribution of radiation intensity

$$I_i(\psi_y, \psi_z) = \frac{c}{4\pi} R^2 E_i^2, \quad i = \theta, \varphi$$

103 using formula for the field components (9) and (10). We put  $\eta = 10^{-2}$ ,  $\gamma = 100$ ,  
104  $\tilde{\alpha} = 10^{-2}$ .

105 Polarization components of radiation are shown in the Figs 2 and 3. Axes  $Y$  and  $Z$   
106 correspond to  $\psi_y$  and  $\psi_z$  respectively. All figures are plotted for interval  $-1 < \psi_y < 1$ ,  
 $-1 < \psi_z < 1$ .

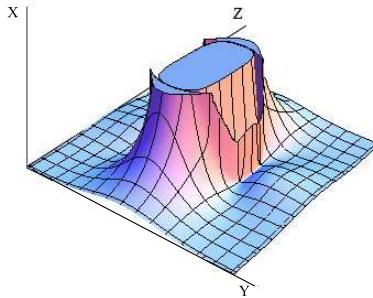


Figure 2: Polar component of radiation intensity  $I_\theta$ .

107  
108 We see that the main part of radiation intensity is the radiation of the channeling  
109 particle which in our case is represented by radiation of an harmonic oscillator moving  
110 with constant velocity. Radiation of this source is investigated in details in many papers  
111 (see for example [9] and references therein). Radiation emitted by the vibrating atoms  
112 is represented as a sharp peak at definite angle  $\psi$ . The angular distribution of total  
113 radiation intensity  $I = I_\theta + I_\varphi$  is depicted in Fig. 4.

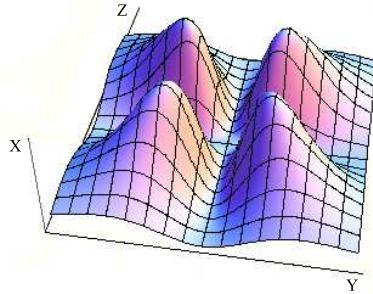


Figure 3: Azimuthal component of radiation intensity  $I_\varphi$ .

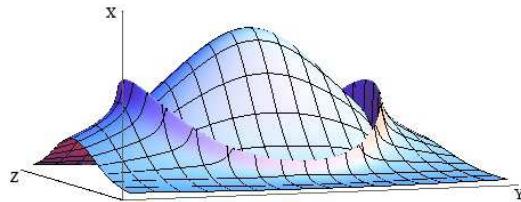


Figure 4: Sum of polarization components.

#### 114 4. Conclusion

115 At first glance, radiation of atoms excited by a channeling particle and vibrating in  
 116 the points of crystal lattice is negligible small because of small amplitude and small  
 117 frequency of vibrations, which is a consequence of relatively great mass of atoms. But  
 118 if we take into account that great number of atoms are involved in coherent oscillations  
 119 by the channeling particle, we get sufficient amplification of the intensity of radiation.  
 120 Derived expressions show that this kind of radiation can be comparable in magnitude with  
 121 radiation of the channeling particle itself. It is shown that the vibrating atoms produce  
 122 radiation field which results in a sharp peak at certain directions on the background of  
 123 radiation of the channeling particle. This effect can be observed at channeling of heavy  
 124 relativistic particles such as protons or ions. Very simplified model of crystal lattice was  
 125 used just to show the essence of the phenomenon. Many details have not been taken into  
 126 account.

127 For example, this paper has not reviewed thermal vibrations of atoms. These can be  
 128 taken into account by adding random oscillations to the regular harmonic oscillations.  
 129 This will cause additional background in intensity of radiation. Evidently, the momentum  
 130 inhomogeneity in a beam of the channeling particles will broaden the resonant peak of  
 131 radiation shown in Figs 2 and 4.

#### 132 Acknowledgement

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